Dynamical Casimir effect via time-dependent conductivity in the MIR experiment

Diego A. R. Dalvit
Theoretical Division-Los Alamos National Laboratory

Reference: M. Crocce, D.D, F. Lombardo and F. Mazzitelli, quant-ph/0404135

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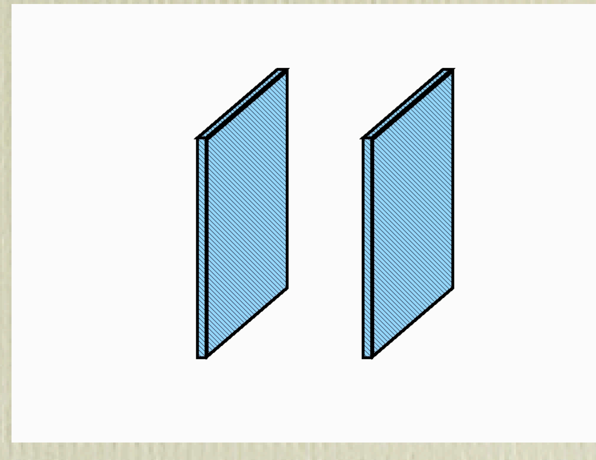
Ideas That Change the World

Non trivial structure
of quantum vacuum → Dynamical Casimir effect

- **Resonant photon creation in oscillating high-Q cavities**

Very high oscillation frequencies $\Omega_{\rm mech} \simeq {\rm GHz}$

Too high to be achieved with a purely mechanical oscillation



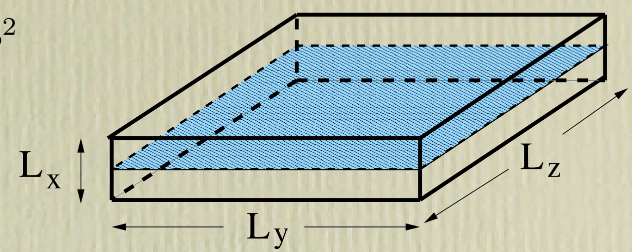
- **MIR experiment**

Ultra short laser pulses periodically irradiated on a semiconductor slab

Effective microwave mirror swichted on and off at very short intervals of time

The model

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{V(t)}{2}\delta(x - L_x/2)\phi^2$$



- Eq. of motion $(\nabla^2 - \partial_t^2)\phi = V(t)\delta(x - L_x/2)\phi$
- Boundary conditions $\text{disc}\ \phi = 0$

$$\text{disc}\ \partial_x\phi = V(t)\phi(x = L_x/2, t)$$

▷ Time-dependent conductivity

$$\boxed{V(t)} \qquad \begin{cases} V \to 0 & \text{‘transparent’ material} \\ V \to \infty & \text{perfect conductor} \end{cases}$$

▷ Electromagnetic analogue

[Barton+Calogeracos, Ann. Phys. 238, 227 (1995)]

Plane-polarized electromagnetic radiation propagating normally to an infinitesimally thin plasma sheet

$$E_y = -\partial_t A_y \qquad B_z = (\nabla\times\mathbf{A})_z = \partial_x A_y$$

- Eq. of motion $(\partial_x^2 - \partial_t^2)A_y = 0$ $\qquad m\,\partial_t^2\eta = -e\,\partial_t A_y(x = L_x/2)$

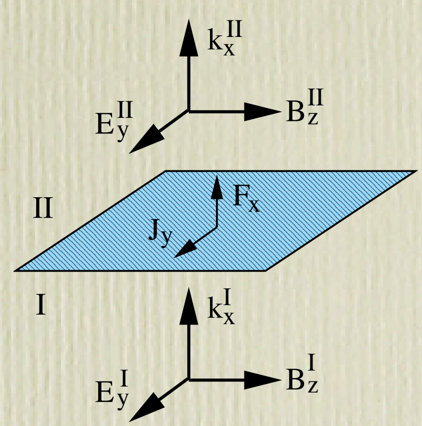
Lateral displacement of charge carriers

- Boundary conditions

$$\text{disc}\ A_y = 0$$

$$\text{disc}\ \partial_x A_y = -4\pi j_y = -4\pi n_s\, e\, \partial_t \eta$$

Surface current density

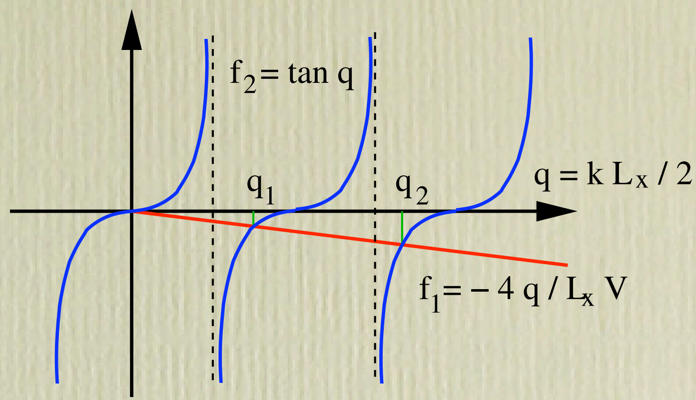


$$\boxed{\phi \leftrightarrow A_y/\sqrt{4\pi}} \qquad \boxed{V \leftrightarrow 4\pi\, n_s\, e^2/m}$$

Two sets of solutions:

- $\varphi_{\mathbf{m}}(\mathbf{x})$ have a node at $x = L_x/2$ and do not see $V(t)$
- $\psi_{\mathbf{m}}(\mathbf{x},t) = \sqrt{\frac{2}{L_x}}\sin(k_{m_x}(t)x)\frac{2}{\sqrt{L_yL_z}}\sin\left(\frac{\pi m_y y}{L_y}\right)\sin\left(\frac{\pi m_z z}{L_z}\right)$

$$-\frac{2k_{m_x}(t)}{V(t)} = \tan\left(\frac{k_{m_x}(t)\,L_x}{2}\right)$$



For $t \le 0$ the semiconductor slab is not irradiated, so $V(t\le 0) = V_0$

$$u_{\mathbf{m}}(\mathbf{x},t) = \frac{e^{-i\tilde{\omega}_{\mathbf{m}}t}}{\sqrt{2\tilde{\omega}_{\mathbf{m}}}}\psi_{\mathbf{m}}(\mathbf{x},0) \qquad \tilde{\omega}_{\mathbf{m}}^2 = (k^0_{m_x})^2 + \left(\frac{\pi m_y}{L_y}\right)^2 + \left(\frac{\pi m_z}{L_z}\right)^2$$

For $t \ge 0$ the slab is irradiated, so $V \to V(t)$ and $k_{m_x} \to k_{m_x}(t)$

Expansion in instantaneous modes: $u_{\mathbf{s}}(\mathbf{x},t>0) = \sum_{\mathbf{m}} P^{(\mathbf{s})}_{\mathbf{m}}(t)\,\psi_{\mathbf{m}}(\mathbf{x},t)$

$$\ddot{P}^{(\mathbf{s})}_{\mathbf{n}} + \omega^2_{\mathbf{n}}(t)P^{(\mathbf{s})}_{\mathbf{n}} = -\sum_{\mathbf{m}}\left[\left(2\dot{P}^{(\mathbf{s})}_{\mathbf{m}}\dot{k}_{m_x} + P^{(\mathbf{s})}_{\mathbf{m}}\ddot{k}_{m_x}\right)g^{(A)}_{\mathbf{mn}} + P^{(\mathbf{s})}_{\mathbf{m}}\dot{k}^2_{m_x}g^{(B)}_{\mathbf{mn}}\right]$$

simple functions of $\psi(\mathbf{x},t)$

Resonant photon creation

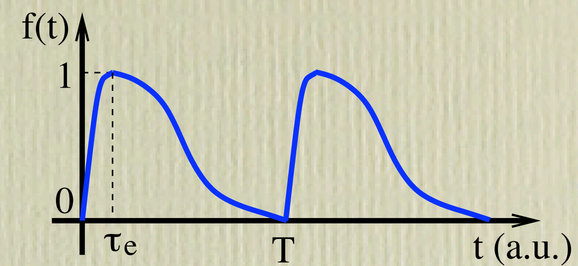
We focus on resonant effects induced by periodic oscillations in the conductivity

$$V(t) = V_0 + (V_{\max} - V_0)\, f(t)$$

$$f(0) = 0$$

$$f(\tau_e) = 1$$

$$f(t) = f_0 + \sum_{j=1}^{\infty} f_j \cos(\Omega_j t + c_j) \qquad \Omega_j = j\,\frac{2\pi}{T}$$



- **Perturbation theory:** When $V_0 L_x \gg V_{\max}/V_0 > 1$ large changes in the conductivity induce small changes in the frequencies of the modes. We employ perturbation in ϵ_n

$$k_n(t) = k_n^0 (1 + \epsilon_n f(t)) \qquad \epsilon_n = \frac{V_{\max} - V_0}{L_x (k_n^0)^2 + V_0 \left(1 + \frac{V_0 L_x}{4}\right)} \ll 1$$

- The modes are a set of coupled harmonic oscillators with periodic frequencies and couplings

$$\ddot{P}_{\mathbf{n}}^{(\mathbf{s})} + \tilde{\omega}_{\mathbf{n}}^2 P_{\mathbf{n}}^{(\mathbf{s})} = -2\epsilon_n (k_n^0)^2 (f - f_0) P_{\mathbf{n}}^{(\mathbf{s})} - \sum_{\mathbf{m}} \left[2\dot{P}_{\mathbf{m}}^{(\mathbf{s})} \dot{f} + P_{\mathbf{m}}^{(\mathbf{s})} \ddot{f} \right] \epsilon_m\, k_m^0\, g_{\mathbf{mn}}^{(A)} + \mathcal{O}(\epsilon^2)$$

Similar to eqns for scalar field modes in 3D cavity with oscillating boundary

is a renormalized frequency, $\tilde{\omega}_{\mathbf{n}}^2 = (\tilde{k}_n^0)^2 + (\pi n_y / L_y)^2 + (\pi n_z / L_z)^2 \qquad \tilde{k}_n^0 \equiv k_n^0 (1 + \epsilon_n f_0)$

- **Parametric resonance:** $\tilde{\omega}_{\mathbf{n}} \leftrightarrow \Omega_j$

- In the resonant case, a naive perturbative solution of the mode equations in powers of ϵ_n breaks down after a short amount of time, of order $\epsilon_n^{-1}\,\Omega_j^{-1}$

- Multiple scale analysis
  - resummation of the perturbative series
  - solution valid for longer times, of order $\epsilon_n^{-2}\,\Omega_j^{-1}$

new time scale: $\tau_n \equiv \epsilon_n\, t$

$$P_{\mathbf{n}}^{(\mathbf{s})}(t) = P_{\mathbf{n}}^{(\mathbf{s})(0)}(t,\tau_n) + \epsilon_n P_{\mathbf{n}}^{(\mathbf{s})(1)}(t,\tau_n) + \mathcal{O}(\epsilon_n^2)$$

- zeroth order: $P_{\mathbf{n}}^{(\mathbf{s})(0)} = A_{\mathbf{n}}^{(\mathbf{s})}(\tau_n)e^{i\tilde\omega_{\mathbf{n}}t} + B_{\mathbf{n}}^{(\mathbf{s})}(\tau_n)e^{-i\tilde\omega_{\mathbf{n}}t}$

- first order: $\partial_t^2 P_{\mathbf{n}}^{(\mathbf{s})(1)} + \tilde\omega_{\mathbf{n}}^2 P_{\mathbf{n}}^{(\mathbf{s})(1)} = -2\partial^2_{t\tau_n} P_{\mathbf{n}}^{(\mathbf{s})(0)} - 2(k_n^0)^2(f-f_0)P_{\mathbf{n}}^{(\mathbf{s})(0)} - \sum_{\mathbf{m}} \frac{\epsilon_m}{\epsilon_n} g^{(A)}_{\mathbf{mn}} \left[2\partial_t P_{\mathbf{m}}^{(\mathbf{s})(0)} k_m^0 \dot f + P_{\mathbf{m}}^{(\mathbf{s})(0)} k_m^0 \ddot f\right]$

  - Key idea of MSA: avoid secularities by imposing that any term $e^{\pm i\tilde\omega_{\mathbf{n}}t}$ in the RHS vanishes

$$\Omega_j = 2\,\tilde\omega_{\mathbf{n}}$$

$$\Omega_j = |\tilde\omega_{\mathbf{n}} \pm \tilde\omega_{\mathbf{m}}|$$

$$\frac{dA_{\mathbf{n}}^{(\mathbf{s})}}{d\tau_n} = 2\sum_j f_j \left\{ -\frac{(k_n^0)^2}{4i\tilde\omega_{\mathbf{n}}} B_{\mathbf{n}}^{(\mathbf{s})} e^{ic_j}\delta(2\tilde\omega_{\mathbf{n}} - \Omega_j) - \sum_{\mathbf{m}} \frac{\epsilon_m}{\epsilon_n}\frac{\Omega_j}{4i\tilde\omega_{\mathbf{n}}} g^{(A)}_{\mathbf{mn}}\, k_m^0\, [(-\frac{\Omega_j}{2} - \tilde\omega_{\mathbf{m}})A_{\mathbf{m}}^{(\mathbf{s})}e^{ic_j}\delta(\tilde\omega_{\mathbf{n}} - \tilde\omega_{\mathbf{m}} - \Omega_j) + \right.$$
$$\left. (-\frac{\Omega_j}{2} + \tilde\omega_{\mathbf{m}})A_{\mathbf{m}}^{(\mathbf{s})}e^{-ic_j}\delta(\tilde\omega_{\mathbf{n}} - \tilde\omega_{\mathbf{m}} + \Omega_j) + (-\frac{\Omega_j}{2} + \tilde\omega_{\mathbf{m}})B_{\mathbf{m}}^{(\mathbf{s})}e^{ic_j}\delta(\tilde\omega_{\mathbf{n}} + \tilde\omega_{\mathbf{m}} - \Omega_j)] \right\}$$

$$\frac{dB_{\mathbf{n}}^{(\mathbf{s})}}{d\tau_n} = (\mathrm{RHS})^\star \ \text{with}\ A_{\mathbf{n}}^{(\mathbf{s})} \leftrightarrow B_{\mathbf{n}}^{(\mathbf{s})}$$

Resonance conditions:

$$\Omega_j = 2\,\tilde{\omega}_{\mathbf{n}}$$

$$\Omega_j = |\tilde{\omega}_{\mathbf{n}} \pm \tilde{\omega}_{\mathbf{m}}|$$

- The eigenfrequencies $\tilde{\omega}_{\mathbf{n}}$ are not equidistant
  In general, when $(j, \mathbf{m}, \mathbf{n})$ satisfy RC, then $(j', \mathbf{m}', \mathbf{n}')$ do not

$\rightarrow$ single Fourier mode $f(t) = f_0 + f_j \cos(\Omega_j t + c_j)$

Parametric resonance case: $\Omega_j = 2\,\tilde{\omega}_{\mathbf{n}}$

- In general, there will be no mode coupling $\rightarrow$ $g^{(A)}_{\mathbf{mn}} = 0$
- $$\frac{dA^{(\mathbf{s})}_{\mathbf{n}}}{d\tau_n} = i\frac{(k_n^0)^2 f_j e^{ic_j}}{\Omega_j} B^{(\mathbf{s})}_{\mathbf{n}} \qquad \frac{dB^{(\mathbf{s})}_{\mathbf{n}}}{d\tau_n} = -i\frac{(k_n^0)^2 f_j e^{-ic_j}}{\Omega_j} A^{(\mathbf{s})}_{\mathbf{n}}$$
- Mean number of created photon with frequency $\tilde{\omega}_{\mathbf{n}} = \Omega_j/2$

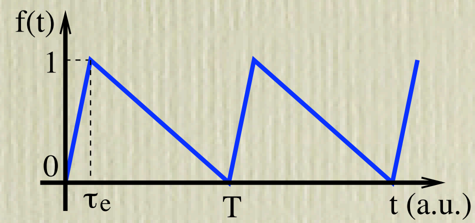
$$\langle \mathcal{N}_{\mathbf{n}}(t)\rangle = \sum_{\mathbf{s}} 2\tilde{\omega}_{\mathbf{n}} |A^{(\mathbf{s})}_{\mathbf{n}}(t)|^2 \approx \sinh^2\left(\frac{(k_n^0)^2 f_j}{\Omega_j}\epsilon_n t\right)$$

exponential growth at a rate $r_{\rm cond} = 2(k_n^0)^2 f_j \epsilon_n/\Omega_j$

Numerical estimations

- Typical values of the conductivity $V(t) = \frac{4\pi e^2}{m} n_s(t)$
  - Slab not illuminated ➡ semiconductor ➡ $V_0 = 10^8\mathrm{m}^{-1} - 10^{13}\mathrm{m}^{-1}$
  - Slab illuminated ➡ good conductor ➡ $V_{\max} = 10^{16}\mathrm{m}^{-1}$
- Small parameter ϵ_n : $10^{-8} \le \epsilon_n \le 10^{-2}$ for a cavity of size $L_x \simeq 10^{-2}\mathrm{m}$
- Profile example for the conductivity: linear ramps $V(t) = V_0 + (V_{\max} - V_0) f(t)$

$$f_j = \frac{1}{\pi j (1-\tau_e/T)} \frac{\sin(\pi j \tau_e/T)}{\pi j \tau_e/T} \approx \begin{cases} 1/\pi j & \text{if } \Omega_j \tau_e \ll 1 \\ T/\tau_e j^2 \pi^2 & \text{if } \Omega_j \tau_e \gg 1 \end{cases}$$



- Rate of photon creation: $r_{\mathrm{cond}} = \epsilon_n/T$ for $\Omega_j \tau_e \ll 1$ and $L_y, L_z \gg L_x$. It is independent of j
- Resonant condition: $\Omega_j = 2\pi j/T \approx \mathrm{GHz}$ It can be achieved with low values of $j \in [1, 10]$ with femtosecond lasers w/ repetition freq. $2\pi/T \approx 100\mathrm{MHz}$
- Excitation time $\tau_e = 10^{-12}\mathrm{sec}$ ➡ $\Omega_j \tau_e \ll 1$ ➡ $1\mathrm{Hz} \le r_{\mathrm{cond}} \le 10^6\mathrm{Hz}$

Comparison with the oscillating mirror:

- Typical photoproduction rates for mechanically oscillating mirrors

$$r_{\rm mov} \approx \epsilon_{\rm mov}/T_{\rm mov}$$

$$T_{\rm mov} = 10^{-9}{\rm sec}$$
$$\epsilon_{\rm mov} < 3\times 10^{-8}$$

- Ratio of photo-production rates

$$\frac{r_{\rm cond}}{r_{\rm mov}} = \frac{\epsilon_n}{\epsilon_{\rm mov}}\frac{T_{\rm mov}}{T} = 10^6\frac{T_{\rm mov}}{T} \gg 1$$

$$\epsilon_n \approx 10^{-2}$$
$$\epsilon_{\rm mov} \approx 10^{-8}$$
$$2\pi/T \approx {\rm MHz}$$

- Detuning

In order to have resonant effect, the external frequency must be tuned with the frequency of the resonant mode with a high accuracy.

- Moving mirror case: detuning $\Delta\Omega_{\rm mov}$ ➡ $\Delta\Omega_{\rm mov}/\Omega_{\rm mov} < \epsilon_{\rm mov}$
- MIR experiment case: detuning $\Delta\Omega_j$ ➡ $\Delta\Omega_j/\Omega_j < \epsilon_n$

Since $\epsilon_n \gg \epsilon_{\rm mov}$ fine tuning is much less severe in the MIR experiment

Summary

- Toy scalar model to mimic photon creation by time-dependent, periodical changes in the conductivity

- For changes in the conductivity of up to six orders of magnitude, the modes of the field oscillate with small amplitudes

- Due to the short excitation time of the semiconductor $(\tau_e/T \ll 1)$ it should be possible to tune a cavity mode with a frequency of a high j Fourier harmonic of the time-dependent conductivity

- As long as $j\pi\tau_e/T \ll 1$ it should be possible to produce resonant effects with ultra-short pulses with repetition frequency well below the GHz range

- Advantages: much faster photo-production rates and milder fine tuning problems